RESISTANCE MODEL FOR HELICAL MCGS

P.T. Tracy
Tracy Physical Sciences, Huntsville, AL
L.L. Altgilbers
U.S. Army Space and Missile Defense Command, Huntsville, AL 35807

ABSTRACT
A semi-empirical computational model describing the design, operation, and time-dependent electrical output of magnetocumulative generators (MCGs) has been presented in previous publications by the authors. This model has been shown to successfully predict the behavior of a number of helical generators developed by laboratories around the world over a number of years. This paper is an initial attempt to describe the physics involved in the time-dependent behavior of the electrical resistance during the operation of these generators. Understanding this parameter is important since it appears to be dependent on the physical dimensions of the generator, the detonation velocity, and the plasma properties of shock heated air. The early-time decrease in resistance appears to be related to the rate at which the plasma is formed, while the late-time increase in resistance is related to the transport properties of the plasma.

Keywords: Magnetocumulative generator, resistance model, Helical MCG

I. INTRODUCTION

A magnetocumulative generator (MCG) is a device that converts the chemical energy of explosives into electrical energy. This is accomplished by locating an explosively loaded armature (or liner) coaxially with a solenoid called the stator. The stator is activated by a seed current from an external power source. When peak current is flowing through the stator, i.e., peak magnetic field within the stator, the explosive in the armature is detonated and the metallic armature surface starts to expand. As the detonation wave proceeds down the length of the armature, the coils of the solenoid are sequentially shorted by the expanding armature. This process leads to a time dependent amplification of the output current from the solenoid into a load.

Magnetocumulative generators were first developed independently in the United States and the Former Soviet Union in the early 1950s. Since that time, a number of other countries have started research programs and there is, at present, several significant research and development programs. One of the most significant programs in the United States is that of Texas Tech University to understand the basic physics and the limitations of MCGs.

A semi-empirical model for helical MCGs is presented in Ref. 1. This model is physics based and was developed to describe the physical processes that take place in the MCG. It was used to successfully describe the operation of a number of different helical MCGs operating under a variety of conditions. It is semi-empirical in that it is based on a single adjustable parameter.

One important characteristic of the helical MCG is its time dependent electrical resistance. An empirical model was developed to describe this resistance as a function of the generator's properties. This paper is an initial attempt to describe those physical processes that affect this electrical resistance.

A brief discussion of the mathematical model for the helical MCG is presented in the next section. This is followed by a detailed discussion of its resistance.

II. MATHEMATICAL MODEL FOR HELICAL MCGS

The circuit equation for an MCG is:

$$\frac{d(LI)}{dt} + RI = 0, \qquad (1)$$

where L and R are the time dependent generator inductance and resistance, respectively, I is the electric current, and t is time. The detonation is initiated at $t = 0$. It is assumed that the inductance and resistance behave exponentially:

$$L = L_0 e^{-t/\tau} \qquad (2)$$

$$R = R_0 e^{-t/\tau}\ , \qquad (3)$$

where L_0 and R_0 are the inductance and resistance just prior to the detonation of the explosive and τ is a characteristic time. This characteristic time depends on the type of explosive used and the dimensions of the device.

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A semi-empirical computational model describing the design, operation, and time-dependent electrical output of magnetocumulative generators (MCGs) has been presented in previous publications by the authors. This model has been shown to successfully predict the behavior of a number of helical generators developed by laboratories around the world over a number of years. This paper is an initial attempt to describe the physics involved in the time-dependent behavior of the electrical resistance during the operation of these generators. Understanding this parameter is important since it appears to be dependent on the physical dimensions of the generator, the detonation velocity, and the plasma properties of shock heated air. The early-time decrease in resistance appears to be related to the rate at which the plasma is formed, while the late-time increase in resistance is related to the transport properties of the plasma.

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The value of τ for the Los Alamos Mark IX generator was determined empirically to be 48 μs.

The solution of Eq. (1), using Eqs. (2) and (3), is:

$$I = I_0 \exp\left(\left(1 - \frac{R_0 \tau}{L_0}\right)\frac{t}{\tau}\right), \qquad (4)$$

where I_0 is the seed current. Equation (4) represents the behavior of the generator from time $t = 0$ to a time t_0 at which the detonation process ends and the current peaks and starts to decrease.

III. MCG RESISTANCE MODEL

The resistance can be written in the form:

$$R = \frac{l}{\sigma A_R}, \qquad (5)$$

where l is the length of the solenoid, σ is the electrical conductivity, and A_R is the annular area between the outer surface of the armature and the inner surface of the stator. As the detonation front proceeds down the armature and the armature expands to make contact with the solenoid, the length, l, decreases:

$$\frac{dl}{dt} = -\frac{l}{\tau} \qquad (6)$$

and

$$l = l_0 e^{-t/\tau}, \qquad (7)$$

where l_o is the initial length of the stator. Thus,

$$R = R_0 e^{-t/\tau}, \qquad (8)$$

where

$$R_0 = \frac{l_0}{\sigma A_R}. \qquad (9)$$

This corresponds to Eq. (3).

It was determined empirically [1], that

$$R_0 = \frac{91 l_a}{I_0}, \qquad (10)$$

where l_a is the length of the armature. The constant, 91, has the units of electric field, V/m. The value of R_0 for the Mark IX generator was calculated to be 3.15 x 10^{-4} Ω.

Relating Eqs. (9) and (10), it is found that the electrical conductivity of the air in the space between the armature and the stator is:

$$\sigma = \frac{l_0 I_0}{91 l_a A_R} \text{ mho/m.} \qquad (11)$$

The area, A_R, is represented by:

$$A_R = \pi\left(r_s^{\,2} - r_a^{\,2}\right), \qquad (12)$$

where r_s is the radius of the stator and r_a is the radius of the armature. The values of the various parameters in Eqs. (11) and (12) for the Mark IX generator are [2]: r_s = 17.8 cm, r_a = 8.65 cm, l_0 = 1.118 m, l_a = 1.73 m, and I_0 = 0.5 MA. Using these parameter values, the electrical conductivity is calculated to be σ = 4.67 x 10^4 mho/m.

The electrical conductivity can also be written as:

$$\sigma = \frac{n_e e^2}{m_e \nu}, \qquad (13)$$

where n_e is the free electron density, e is the electron charge, m_e is the electron mass, and ν is the electron collision frequency. This expression assumes that the electron mobility is given by:

$$\mu = \frac{e}{m_e \nu}. \qquad (14)$$

Writing the electron collision frequency as

$$\nu = n_a \sigma_c \left(\frac{3kT}{m_e}\right)^{1/2} \qquad (15)$$

and using Eq. (13), the initial free electron density is found to be:

$$n_e = \frac{\sigma n_a \sigma_c}{e^2}(3 m_e k T)^{1/2}, \qquad (16)$$

where n_a is the ambient atmospheric particle density, σ_c is the electron collision cross-section with air particles, k is the Boltzmann constant, and T is the temperature. In writing Eq. (16), it was assumed that the degree of ionization is relatively low and that the electrons are in thermal equilibrium with the ambient gas in the MCG. In most cases, the gas is air.

As a numerical example, consider the ambient gas to be air at sea level density (n_a = 2.69 x 10^{25} m^{-3}) and a temperature of 288.76^{0}K. For a conductivity of 4.67 x 10^4 mho/m, it is found using Eqs. (16) and an electron collision cross-section of 9.4 x 10^{-20} m^2, that the electron density is 4.82 x 10^{23} m^{-3}. This corresponds to a 1.8% degree of ionization.

As pointed out in the previous section, the resistance decreases exponentially out to a time t_0. This time is represented by the formula:

$$t_0 = \frac{l_a}{v_d}, \qquad (17)$$

where v_d is the detonation velocity. The detonation velocity depends on the explosive used. As an example, the explosive used in the Mark IX was PBX9501, which has a detonation velocity of 8.83 km/s [2]. Thus,

$t_0 = 1.96 \times 10^{-4}$ s .

For times greater than t_0, it has been found, empirically, that Eq. (4) must be modified to the following form:

$$I = I_0 \exp\left\{\left[1 - \frac{R_0 \tau}{L_0} e^{b(t-t_0)}\right]\frac{t}{\tau}\right\} \qquad (18)$$

for $t \geq t_0$, where $b \equiv 1.8 \times 10^5$ s^{-1}. The inverse of b is a second characteristic time, 5.56 µs. It should be noted that Eq. (18) has a peak value at a time slightly greater than t_0. As an example, again using the Mark IX generator, the peak value of Eq. (18) occurs at $t \approx 212$ µs, which is approximately 16 µs after the end of the detonation, t_0.

The form of Eq. (18) indicates that after $t = t_0$, the resistance has the form:

$$R = R_0 \exp\left[b(t - t_0) - \frac{t}{\tau}\right] \qquad (19)$$

for $t \geq t_0$.

For the Mark IX generator, the resistance has decreased from its initial value of 3.15 x 10^{-4} Ohms to $5.31 \times 10^{-6}\ \Omega$ at $t = t_0$. At $t =$ 212 µs, where the current reaches a peak value, the resistance has increased to 6.78 x 10^{-5} Ohms. The resistance increases to its initial value at:

$$t = \frac{t_0}{1 - \frac{1}{b\tau}} = 2.22 \times 10^{-4}\ \text{s}. \qquad (20)$$

Thus, the resistance has increased by a factor of 59.3 in a period of 26 µs following the end of the detonation and continues to increase very rapidly.

IV. SUMMARY

This paper presents an initial effort to describe the physical processes involved in the time varying resistance of an MCG. The variation in the resistance was shown to occur over three separate time regimes. The first time period is that following the onset of the seed current, but prior to the detonation of the explosive charge. The second time period is the time it takes for the detonation wave to travel the length of the explosive loaded armature. The third, and final, period is the time following the end of the detonation to the time at which the current in the load has been reduced to zero.

During the first period of time, the resistance reaches a constant value that is determined by the amplitude of the seed current, the dimensions of the generator, and the surrounding medium (because the factor 91 has the units of electric field and because of the collision frequency). In most cases studied thus far, the surrounding medium has been air.

The resistance during the second time period decreases rapidly with a characteristic time that depends on the dimensions of the generator and the type of explosive used. The rate of decrease does not appear to be dependent on the surrounding medium.

The final period of time, following the end of the detonation process, the resistance rapidly increases. The characteristic time for the increase of the resistance is 5.56 µs and is independent of the generator and explosive characteristics. This would seem to indicate that it is dependent on the surrounding medium.

Future work on MCGs will include an effort to determine the nature of the dependence on the medium in which the interaction is taking place. This work should include MCG operation in a vacuum.

V. REFERENCES

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